# Lecture = 1

## Unit = I

## Discrete structure
OR
## Discrete mathmatics.

Discrete mathmatics in the part of mathmatics devoted to the study of discrete objects.

Discrete objects means the are

Countable
separate
distict.

Example :- Integers (whole numbers)

Houses
Bus
People etc

Why do we study Discrete mathematics.

Topics in discrete mathematics are :-

$Q = \{x \mid x = \frac{p}{q} ;\ p, q \in Z,\ q \neq 0\}$

Any number which can be written as a fraction

ex - $\frac{-2}{3}$, $-0.4$, $5.2$, $0.001$

$Q = \{1.5, 2.6, -3.8, 12, \ldots\}$

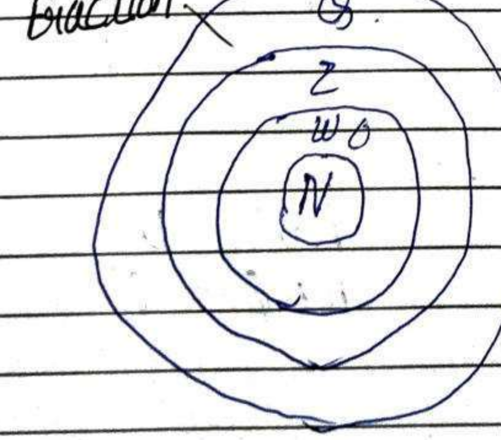
Irrational numbers :- Any decimal number which can't be written as a fraction (not rational numbers)

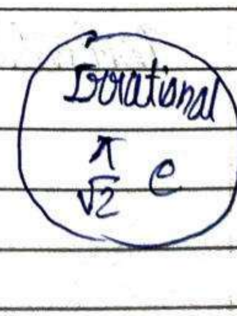
A non - terminating & non-repeating decimal

Example :- $\pi = 3.141592\ldots$
$\sqrt{2} = 1.41421\ldots$

R = set of all real numbers. [set of rational & irrational numbers]

$R = \{47.3, \pi, \sqrt{2}, -2.5, 12 \ldots\}$



From ① & ②

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Hence proved

## Distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A = \{1, 2\}$ $B = \{2, 3\}$

let $x^3 \in A \cup (B \cap C)$ $x = 3$ $C = \{1, 3, 1\}$

$x^3 \in A$ or $x^3 \in (B \cap C)$ $B \cap C = \{3\}$

$A \cup (B \cap C) = \{1, 2, 3\}$

$x \in A$ or $(x \in B$ and $x \in C)$ $\uparrow x$

$(x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$x \in (A \cup B) \cap x \in (A \cup C)$

$x \in (A \cup B) \cap (A \cup C)$

## Distributive law

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof Let $x \in A \cup (B \cup C)$ — ①

$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

(using distributive law for logical expresion)

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

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## Numericals on symmetic relation

① $A = \{1, 2, 3, 4\}$
$R = \{(1,2)\ (2,1)\ (3,3)\ (4,4)\}$

Sol
$\overset{a\ b}{(1,2)} \longrightarrow \overset{b\ a}{(2,1)}$ if $(a,b) \in R$ Then $(b,a) \in R$
$(3,3) \longrightarrow (3,3)$
$(4,4) \longrightarrow (4,4)$

given relation is symmetric

2 $A = Z \Rightarrow A = \{0, \pm 1, \pm 2, \ldots\}$
$R = \{(a,b) \mid 3 \text{ divides } a-b\}$

Relation R is symmetric as if $(a-b)$ will be divided by 3 then $(b-a)$ will also divided by 3.

## Numericals on Transitive Relation  L = 22

$A = Z \Rightarrow A = \{0, \pm 1, \pm 2, \pm 3 \ldots\}$
$R = \{(a,b) \mid a > b\}$

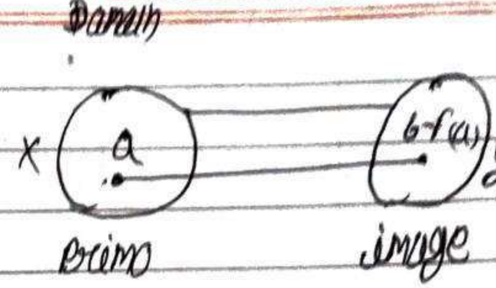
Sol
Transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

i.e if $a > b$ and $b > c$ then $a > c$

ex $6 > 3$ and $3 > 2$ then $6 > 2$

$\therefore$ R is transitive.

# Function




The relationship from the element of one set x to element of another set y is defined as function or mapping

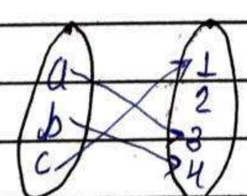
It is ~~repre~~ represented a $\quad$ x, y - non empty set

$f : x \Rightarrow y \qquad f(x) = y \quad x \in X, \; y \in Y$

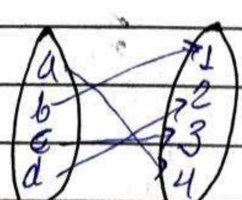
For the function f, x is the domain or preimage & y is the codomain of image.

Function f is a relation on x & y such that for each $x \in x$, there exist a unique $y \in y$ such that $(x, y) \in f$

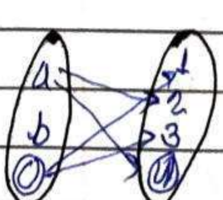
Examples



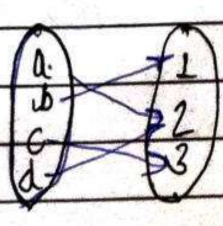

One to one function




one to one onto $f^n$




Not a valid funtion




Many to one $f^n$ onto $f^n$

$(N, +)$ - is an algebraic structures

$(N, -)$ is not an algebraic structures as $-$ is not a binary operation on set $N$.

$(Z, +)$ $(Z, -)$

Group
Rings
Fields

] Algebraic structure

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Properties of ~~A~~ algebraic structure

A property possessed by any operations of an algebraic ~~str~~ structure

1) Associative law

An operations $*$ on a set G is said to be associative or to satisfy the associative law, if

$$(a*b)*c = a*(b*c) \quad \text{where } a, b, c \in G$$

Eg. $G = \{1, 2, 3, 4\}$ $+ \rightarrow$ is associative but $-$ is not associative

2 Commutative law :-

An operations $*$ on a set G is said to be commutative or

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inverse of (ab) [ $a$ - element $\Rightarrow a^{-1}$ inverse]
[ab - element $\Rightarrow (ab)^{-1}$ inverse]

if $(ab)^{-1} = b^{-1}a^{-1}$ that mean
$b^{-1}a^{-1}$ is inverse of ab]

L.H.S $(ab)(b^{-1}a^{-1}) = a.(b.b^{-1})a^{-1} = a.e.a^{-1} = aa^{-1} = e$

R.H.S $(b^{-1}a^{-1})(ab) = b^{-1}(a^{-1}a)b = b^{-1}.e.b = b^{-1}b = e$

L.H.S = R.H.S

Hence $b^{-1}a^{-1}$ is inverse of ab
$(ab)^{-1} = b^{-1}a^{-1}$

(2) $(a^{-1})^{-1} = a$

Proof Let $a \in G$ & $b$ is inverse of $a$.

As we know that

$a * b = e = b * a$

Here $a$ is inverse of $b$ & $b$ is inverse of $a$ [ $b = a^{-1}$ ]

So $b^{-1} = a \Rightarrow (a^{-1})^{-1} = a$
$\Rightarrow$ [As $b = a^{-1}$]

Hence proved

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## Modulo addition

Q Prove that $G = (\{0,1,2,3\}, +_4)$ is a group

Solution

(1) closure property

$0+1=1$
$2+3 = 5-4 = 1$
$1+2 = 3$

$a +_4 b = a+b$ if $a+b<4$
$a +_4 b = a+b-4$ if $a+b >= 4$
$= (a+b) \% 4$

| $+_4$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 2 | 3 | 0 |
| 2 | 2 | 3 | 0 | 1 |
| 3 | 3 | 0 | 1 | 2 |

for all $a+b \in G$ closure property satisfied

2 Associative property

$a +_4 (b +_4 c) = (a +_4 b) +_4 c$
$1 +_4 (2 +_4 3) = (1 +_4 2) +_4 3$
$1 +_4 (5-4) = 3 +_4 3$
$1 +_4 1 = 6-4$
$2 = 2$ Satisfied

3 Identity Element $a +_4 e = a$

$2 +_4 e = 2 \Rightarrow 2 +_4 0 = 2$ — $e=0$
$3 +_4 e = 3 \Rightarrow 3 +_4 0 = 3$

$\therefore e=0$ & $0 \in G$. Satisfied

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let $x \in aH$ , $x \in Ha$

$x = a*h$ — ① $x = h_1 * a$ — ②

from ①&② $a*h = h*a$
Multiply $a^{-1}$ on L.H.S & R.H.S

$(a*h) * a^{-1} = (h_1 * a) * a^{-1}$
$= h_1 * (a*a^{-1})$
$= h_1 * e$
$(a*h) * a^{-1} = h_1$ & $h_1 \in H$

$\therefore (a*h) * a^{-1} = h_1$ & $h_1 \in H$

$\therefore (a*h)*a^{-1} \in H$

or $aha^{-1} \in H$ $\forall h \in H, a \in G$

Let $\underline{x \in aH}$ — ①
$x = a*h = ah$

multiply $a^{-1}$ on both sides

$xa^{-1} = aha^{-1}$, here $aha^{-1} \in H$
(according to
assumpti

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## Ring

Algebraic structure − (G, *) one binary operation − Group

Set + one or more binary Operation

Ring − (R, +, ·) two binary operations

An algebraic structure (R, +, ·) is a Ring if R is a non-empty set & '+' and (·) are binary operations such that

(1) (R, +) is an abelian group — closure property, Associative property, Existence of Identity, Existence of Inverse, commutative law

(2) (R, ·) is a semi group − [Closure, Associative]

(3) It should satisfy left distributive & right distributive law

~~Asso~~ left distributive law $a\cdot(b+c) = a\cdot b + a\cdot c \quad \forall a,b,c \in R$

Right distributive law $(b+c)\cdot a = b\cdot a + c\cdot a$ or $(a+b)\cdot c = a\cdot c + b\cdot c$

(R, +) is an abelian group if [ m, addition, matrix add ]

~~(R, +) is an abelian group~~

II Now we have to check $(Z_3, \times_3)$ is an ablian group

(i) closure property

| $\times_3$ | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 |
| 2 | 0 | 2 | 1 |

$\forall a, b \in Z_3$
$\Rightarrow a \times_3 b \in Z_3$

satistisfied

Associativity :- $a \times_3 (b \times_3 c) = (a \times_3 b) \times_3 c$

$a = 1, b = 0, c = 2$ $\quad 1 \times_3 (0 \times_3 2) = (1 \times_3 0) \times_3 2$
$1 \times_3 0 = 0 \times_3 2$
$0 = 0$

let
$a = 1$ $b = 2$ $c = 2$ $\quad 1 \times_3 (2 \times_3 2) = 1 \times_3 2) \times_3 2$
$1 \times_3 (4 \% 3) = 2 \times_3 2$
$1 \times_3 1 = 4 \% 3$
$1 = 1$

iii Existence of Identity =
$a \times_3 e = e \times_3 a \quad \forall a \in Z_3$

for non zero elements only
$1 \times_3 \_\_\_ = 1 \Rightarrow 1 \times_3 1 = 1$
$2 \times_3 \_\_\_ = 2 \Rightarrow 2 \times_3 1 =$
$\therefore e = 1$

Existence of Inverse:
$a \times_3 a^{-1} = e = a^{-1} \times_3 a$

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## Logical Equivalence :-

Two compound statement P & Q will be logically equivalence (denoted by $P \equiv Q$) when their truth values are exactly same or when their biimplication ($P \leftrightarrow Q$) is a tautology

Example $P \wedge T \equiv P$ $\quad$ $P \vee F \equiv P$

| P | T | $P \wedge T$ |
|---|---|---|
| T | T | T |
| F | T | F |

same truth values

| P | F | $P \vee F$ |
|---|---|---|
| T | F | T |
| F | F | F |

same truth values

T is tautology
F is null set

Most common logical equivalence are

1 Identity law

$P \wedge T \equiv P$

$P \vee F \equiv P$

2 Domination law

$P \wedge F \equiv F$

$P \vee T \equiv T$

6 Negation law

$P \vee \sim P \equiv T$

$P \wedge \sim P \equiv F$

7 Double Negation law

$\sim(\sim P) \equiv P$

$P(x)$: $x+3>x$ domain of discourse - real
find truth value of $\forall x\, P(x)$

$\forall x\, P(x)$, $P(x)$ is true.

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## Normal forms

Normal forms provids an independent test as to whether given compound propositions is a tautology or contradiction & they are also used to check the given compound propositions are logically equivalent or not without using truth table

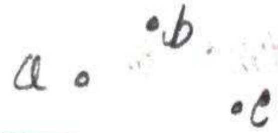
Atom :- It is either a propositional variable or a negated propositional variable,

Example
$P, \sim P, Q, \sim R, \sim Q$

Elementary disjunction

Let $A_1, A_2, \ldots, A_n$ be a list of atoms where $n \geq 1$

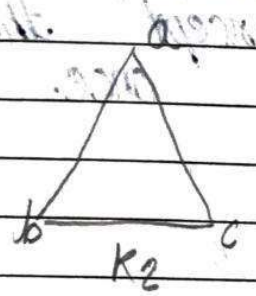
Any list of atoms linked by $\vee$'s (or) count as an elementary disjunction

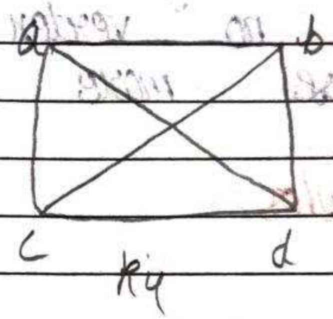


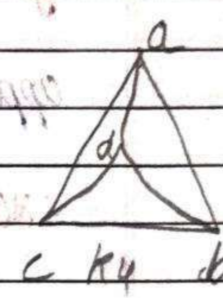

5 **Complete graph** :- A graph in which every pair of vertices has a unique edge is called as complete graph.

OR

A graph is said to be complete if each vertex is connected to every other vertex of a graph.




~~$K_n$ is a complete if each vertex is connected to every other vertex of a graph~~

$K_n$ is a complete graph with 'n' no. of vertices

$K_n$ has exactly $\frac{n(n-1)}{2}$ edges

So the solution is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

Where $a_n^{(h)}$ = solution of associated homogeneous equation

$a_n^{(p)}$ = particular solution

Calculating homogeneous solution

~~So the solution is~~

$$\cancel{a_n = a_n^{(h)} + a_n^{(p)}}$$

Homogeneous eq$^n$ is

$$a_n = 3a_{n-1} + 10a_{n-2}$$

or $a_n - 3a_{n-1} - 10a_{n-2} = 0$

put $a_n = r^n$

$$r^n - 3r^{n-1} - 10r^{n-2} = 0$$

Divide both sides by $r^{n-2}$, we

$$\frac{r^n}{r^{n-2}} - \frac{3r^{n-1}}{r^{n-2}} - \frac{10r^{n-2}}{r^{n-2}} = 0$$

$$r^2 - 3r - 10 = 0$$

Compare eq$^n$ with $ax^2 + bx + c = 0$

$$a = 1, \; b = -3, \; c = -10$$